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Global Exponential Stability of Fuzzy Cellular Neural Networks with Delay

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Abstract : *In this paper, without assuming the boundedness, monotonicity and differentiability of the activation functions, we present new conditions ensuring existence, uniqueness, and global asymptotical stability of the equilibrium point of bidirectional associative memory neural networks with fuzzy logic and time delays. The results are applicable to both symmetric and nonsymmetric interconnection matrices, and all continuous non-monotonic neuron activation functions. Since the criterion is independent of the delays and simplifies the calculation, it is easy to test the conditions of the criterion in practice. An example is given to demonstrate the feasibility of the criterion.*

Keywords *- bidirectional associative memory, neural networks, global asymptotic stability, fuzzy logic, time delays*

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I. Introduction

There are two basic cellular neural networks structures being proposed, namely traditional CNNs and fuzzy CNNs [1]. The latter integrates fuzzy logic into the structure of traditional CNNs. It is known that in hardware implementation, time delays occur due to finite switching speeds of the amplifiers and communication time, lead to an oscillation and furthermore, to instability of networks. Therefore, the study of stability of FCNNs with delays is practically required. In most situations, delays are variable. Some conditions ensuring the globally exponential stability of FCNNs with variable time delays are given in [2], in which differentiability of the delays is assumed and the unbounded delays was not involved. It is more likely there are multiple states, even infinite states affecting the current state. Some results on the stability of neural networks involving time delays are given in [3-8]. However, FCNNs with unbounded delays and variable coefficients are seldom considered. In [9], authors have obtained some results regarding FCNNs with distributed delays, but the global exponential stability of FCNNs with unbounded delay is not studied. In [10], the state estimation problem was studied for the fuzzy cellular neural networks with unbounded distributed delays, but the coefficients of the fuzzy system was assumed to be constant .

In this paper, we study the existence, uniqueness and globally exponential stability of the equilibrium point of FCNNs with variable coefficients, in which activation functions are Lipschitz continuous and there exist unbounded delays. By constructing proper nonlinear integro-differential inequalities, applying M-matrix theory and vector Liapunov function method, we obtain sufficient conditions for existence, uniqueness and globally exponential stability of the equilibrium point of FCNNs with variable coefficients and unbounded delays.

II. Assumption and Lemma

In this paper, u^{T} and A^{T} denote the transpose of a vector u and a matrix A , where $u \in R^n$ and $A \in R^{n\times n}$. $[A]^s$ is defined as $[A]^s = [A^{\mathrm{T}} + A]/2$. $|u|$ denotes the absolute-value vector given by $|u| = (|u_1|, \cdots, |u_n|)^{\mathrm{T}}$, $||u||$ denotes a vector norm defined by $\| u \| = (u_1^2 + \cdots + u_n^2)^{1/2}$. A^{-1} denotes the inverse of A, and $|A|$ denotes absolute-value matrix given by $|A| = (|a_{ij}|)_{n\times n}$, $||A||$ denotes a matrix norm defined by $||A|| = (\max\{\lambda : \lambda$ is an eigenvalue of $A^{\mathrm{T}}A\})^{1/2}$, det (A) denotes the determinant of matrix A. $D = \mathrm{diag}(d_1, \cdots, d_n)$ represents a diagonal matrix. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively.

The dynamical behavior of FCNNs with unbounded and variable time delays can be described by the following nonlinear differential equations:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i(t)x_i(t) + \sum_{j=1}^{n} b_{ij}(t) f_j(x_j(t - \tau_{ij}(t)))$$
$$+\sum_{j=1}^{n} c_{ij}(t)u_j + J_i(t) + \bigwedge_{j=1}^{n} \alpha_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) f_j(x_j(s))ds$$
$$+\bigvee_{j=1}^{n} \beta_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) f_j(x_j(s))ds + \bigwedge_{j=1}^{n} R_{ij}(t)u_j + \bigvee_{j=1}^{n} S_{ij}(t)u_j,\ i=1,2,\cdots n, \tag{1}$$

where x_i is the state of neuron i, $i=1,2,\cdots,n$, and n is the number of neurons; u_i and $J_i(t)$ denote input and bias of the ith neuron, respectively; f_i is the activation function of the ith neuron; $d_i(t)$ is the damping constants, and $d_i(t) > 0$; $b_{ij}(t)$, $c_{ij}(t)$ are elements of feedback template and feedforward template; $\alpha_{ij}(t)$, $\beta_{ij}(t)$, $R_{ij}(t)$ and $S_{ij}(t)$ are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; $\tau_{ij}(t)$ denote the variable time delays. Assume that the delays $\tau_{ij}(t)$ are bounded, continuous with $\tau_{ij}(t) \in [0,\tau]$ for all $t \ge 0$, where τ is a constant, $i, j=1,2,\cdots,n$. $k_{ij}:[0,\infty) \to [0,\infty)$ ($i, j=1,2,\cdots,n$) are piecewise continuous on $[0,\infty)$ and satisfy

$$\int_0^{\infty} \mathrm{e}^{\beta s} k_{ij}(s)ds = p_{ij}(\beta),\ i, j=1,2,\cdots,n,$$

where $p_{ij}(\beta)$ are continuous functions in $[0,\delta)$, $\delta > 0$, and $p_{ij}(0)=1$.

The initial conditions associated with equation (1) are of the form

$$x_i(s) = \phi_i(s),\ -\tau \le s \le 0,\ \phi_i \in C([-\tau,0],R),\ i, j=1,2,\cdots,n$$

where $\phi_i(s)$ is bounded and continuous.

In this paper, we give the following assumptions.

Assumption 1. For system (1), there exit constant numbers d_i, b_{ij}, α_{ij}, β_{ij}, such that

$$d_i = \inf_{t\ge0}\{d_i(t)\},\ b_{ij} = \sup_{t\ge0}\{b_{ij}(t)\},\ \alpha_{ij} = \sup_{t\ge0}\{\alpha_{ij}(t)\},\ \beta_{ij} = \sup_{t\ge0}\{\beta_{ij}(t)\}, i, j=1,2,\cdots,n.$$

Assumption 2. For each $j \in \{1,2,\cdots,n\}$, the activation function $f_j : R \to R$ is globally Lipschitz with Lipschitz constants $L_j > 0$, i.e. $| f_j(x_j) - f_j(y_j) | \le L_j | x_j - y_j |$ for all x_j, y_j.

In the following, we let

$$D = diag(d_1, d_2,\cdots,d_n),\ B = (b_{ij})_{n\times n},\ \alpha = (\alpha_{ij})_{n\times n},$$
$$\beta = (\beta_{ij})_{n\times n}, L = \mathrm{diag}(L_1, L_2,\cdots,L_n).$$

To obtain our results, we give the following definitions and lemmas.

Definition 1. The equilibrium point x^* of (1) is said to be globally exponentially stable, if there exist constant $\lambda > 0$ and $M > 0$ such that

$$\| x(t) - x^*(t) \| \le M \| \phi - x^*(t) \| e^{-\lambda t}$$

for all $t \ge 0$, where $\| \phi - x^* \| = \max_{1\le i\le n}\{ \sup_{s\in[-\tau,0]} | \phi_i(s) - x_i^* | \}$.

Definition 2. A real $n\times n$ matrix $A=(a_{ij})$ is said to be an M-matrix if $a_{ij} \le 0$, $i, j=1,2,\cdots,n$, $i \ne j$, and all successive principal minors of A are positive.

Lemma 1. Let $A=(a_{ij})$ be a matrix with non-positive off-diagonal elements. Then the following statements are equivalent:

(i) A is an M-matrix;

(ii) The real parts of all eigenvalues of A are positive;

(iii) There exists a vector $\xi > 0$, such that $\xi^{\mathrm{T}} A > 0$;

(iv) A is nonsingular and all elements of A^{-1} are nonnegative;

(v) There exists a positive definite $n \times n$ diagonal matrix Q such that matrix $AQ + QA^{\mathrm{T}}$ is positive definite.

Lemma 2. Suppose x and x' are two states of system (1), then

$$|\bigwedge_{j=1}^{n}\alpha_{ij} f_j(x_j) - \bigwedge_{j=1}^{n}\alpha_{ij} f_j(x_j')| \le \sum_{j=1}^{n}|\alpha_{ij}\|f_j(x_j) - f_j(x_j')|$$

$$|\bigvee_{j=1}^{n}\beta_{ij} f_j(x_j) - \bigvee_{j=1}^{n}\beta_{ij} f_j(x_j')| \le \sum_{j=1}^{n}|\beta_{ij}\|f_j(x_j) - f_j(x_j')|$$

Lemma 3. If $H(x) \in C^0$ satisfies the following conditions, then $H(x)$ is a homeomorphism of R^n.

i) $H(x)$ is injective on R^n,

ii) $\lim\limits_{\|x\|\to\infty} \|H(x)\| \to \infty$.

III. Existence of the equilibrium

In the section, we study the existence and uniqueness of the equilibrium point of the system (1). We first study the solutions of the nonlinear map associated with (1) as follows:

$$H_i(x_i) = -d_i(t)x_i + \sum_{j=1}^{n} b_{ij}(t) f_j(x_j)$$
$$+\bigwedge_{j=1}^{n}\alpha_{ij}(t) f_j(x_j) + \bigvee_{j=1}^{n}\beta_{ij}(t) f_j(x_j) + I_i,\ i = 1,2,\cdots n, \tag{2}$$

where $I_i = \sum_{j=1}^{n} c_{ij}(t)u_j + \bigwedge_{j=1}^{n} R_{ij}(t)u_j + \bigvee_{j=1}^{n} S_{ij}(t)u_j + J_i(t)$, $i = 1,2,\cdots n$. We let $H = (H_1, H_2, \cdots, H_n)^{\mathrm{T}}$, $I = (I_1, I_2, \cdots, I_n)^{\mathrm{T}}$.It is well known that the solutions of $H(x) = 0$ are equilibriums in (1). If map $H(x)$ is a homeomorphism on R^n, then there exists a unique point x^* such that $H(x^*) = 0$, i.e., systems (1) have a unique equilibrium x^*.

Theorem 1. If Assumption 1 is satisfied, and $D - (|B| + |\alpha| + |\beta|)L$ is an M matrix, then for each u, system (1) has a unique equilibrium point.

Proof. In order to prove that for every input u, (1) has a unique equilibrium point x^*, it is only to prove that $H(x)$ is a homeomorphism on R^n. In the following, we shall prove that map $H(x)$ is a homeomorphism in two steps.

In the first step, we prove that $H(x)$ is an injective on R^n. For purposes of contradiction, suppose that there exist $x, y \in R^n$ with $x \ne y$, such that $H(x) = H(y)$. From Assumption 1, Assumption 2, Lemma 2 and (2), we get

$$|H_i(x_i) - H_i(y_i)|$$
$$=| -d_i(t)(x_i - y_i) + \sum_{j=1}^{n} b_{ij}(t)[f_j(x_j) - f_j(y_j)]$$
$$+\bigwedge_{j=1}^{n}\alpha_{ij}(t) f_j(x_j) - \bigwedge_{j=1}^{n}\alpha_{ij}(t) f_j(y_j) + \bigvee_{j=1}^{n}\beta_{ij}(t) f_j(x_j) - \bigvee_{j=1}^{n}\beta_{ij}(t) f_j(y_j)|$$
$$\ge d_i(t)|x_i - y_i| - \sum_{j=1}^{n}|b_{ij}(t)||f_j(x_j) - f_j(y_j)|$$
$$-|\bigwedge_{j=1}^{n}\alpha_{ij}(t) f_j(x_j) - \bigwedge_{j=1}^{n}\alpha_{ij}(t) f_j(y_j)| - |\bigvee_{j=1}^{n}\beta_{ij}(t) f_j(x_j) - \bigvee_{j=1}^{n}\beta_{ij}(t) f_j(y_j)|$$

$$\geq d_i \mid x_i - y_i \mid - \sum_{j=1}^{n} \mid b_{ij} \mid \mid f_j(x_j) - f_j(y_j) \mid$$

$$- \sum_{j=1}^{n} \mid \alpha_{ij} \mid \mid f_j(x_j) - f_j(y_j) \mid - \sum_{j=1}^{n} \mid \beta_{ij} \mid \mid f_j(x_j) - f_j(y_j) \mid$$

$$\geq d_i \mid x_i - y_i \mid - \sum_{j=1}^{n} (\mid b_{ij} \mid + \mid \alpha_{ij} \mid + \mid \beta_{ij} \mid) L_j \mid x_j - y_j \mid, \ i = 1, 2, \cdots n. \tag{3}$$

Rewrite the above inequalities (3) in matrix form, we get

$$\mid H(x) - H(y) \mid \geq [D - (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L] \mid x - y \mid. \tag{4}$$

Since $D - (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L$ is an M-matrix, from Lemma 1, $[D - (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L]^{-1}$ is a nonnegative matrix. Thus multiplying both sides of the above inequality by $[D - (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L]^{-1}$ does not change the inequality direction, it comes to

$$\mid x - y \mid \leq [D - (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L]^{-1} \mid H(x) - H(y) \mid.$$

From $H(x) = H(y)$, we have $\mid x - y \mid \leq 0$, so $\mid x - y \mid = 0$, i.e., $x = y$. From the supposition $x \neq y$, thus this is a contradiction. So map $H(x)$ is injective.

In the second step, we prove that $\lim_{\|x\| \to \infty} \| H(x) \| \to \infty$. Let

$$\overline{H}(x) = H(x) - H(0). \tag{5}$$

To prove that $H(x)$ is a homeomorphism, if only suffices to show that $\overline{H}(x)$ is a homeomorphism. Because of $D - (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L$ is an M-matrix, from Lemma 1, there exists a positive define diagonal matrix $T = \mathrm{diag}\{T_1, T_2, \cdots, T_n\}$, such that

$$[T(D - (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L)]^s \geq \varepsilon E_n > 0,$$

So, we have

$$[T(-D + (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L)]^s \leq -\varepsilon E_n < 0, \tag{6}$$

where ε is a sufficiently small positive number and E_n is the identity matrix. From Assumption 1, Assumption 2, Lemma 2 and (6), we get

$$[Tx]^{\mathrm{T}} \overline{H}(x) = [Tx]^{\mathrm{T}} (H(x) - H(0))$$

$$= \sum_{i=1}^{n} T_i \{ -d_i(t) x_i^2 + x_i \sum_{j=1}^{n} b_{ij}(t) [f_j(x_j) - f_j(0)]$$

$$+ x_i [\bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(0)] + x_i [\bigvee_{j=1}^{n} \beta_{ij}(t) f_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(0)]\}$$

$$\leq \sum_{i=1}^{n} T_i \{ -d_i(t) \mid x_i \mid^2 + \mid x_i \mid \sum_{j=1}^{n} \mid b_{ij}(t) \mid \mid f_j(x_j) - f_j(0) \mid$$

$$+ \mid x_i \mid \mid \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(0) \mid + \mid x_i \mid \mid \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(0) \mid \}$$

$$\leq \sum_{i=1}^{n} T_i \{ -d_i \mid x_i \mid^2 + \mid x_i \mid [\sum_{j=1}^{n} \mid b_{ij} \mid \mid f_j(x_j) - f_j(0) \mid$$

$$+ \sum_{j=1}^{n} \mid \alpha_{ij} \mid \mid f_j(x_j) - f_j(0) \mid + \sum_{j=1}^{n} \mid \beta_{ij} \mid \mid f_j(x_j) - f_j(0) \mid]\}$$

$$\leq \sum_{i=1}^{n} T_i \{ -d_i \mid x_i \mid^2 + \mid x_i \mid \sum_{j=1}^{n} (\mid b_{ij} \mid + \mid \alpha_{ij} \mid + \mid \beta_{ij} \mid) L_j \mid x_j \mid \}$$

$$= \mid x \mid^{\mathrm{T}} [T(-D + (\mid B \mid + \mid \alpha \mid + \mid \beta \mid) L)] \mid x \mid$$

$$=|x|^{\mathrm{T}}\ [T(-D+(|B|+|\alpha|+|\beta|)L)]^{\mathrm{T}}\ |x|$$
$$=|x|^{\mathrm{T}}\ [T(-D+(|B|+|\alpha|+|\beta|)L)]^{s}\ |x| \le -\varepsilon \|x\|^{2} \quad (7)$$

Using Schwarz inequality, from (7), we get $\varepsilon \|x\|^2 \le \|T\| \|x\| \|\overline{H}(x)\|$, so $\|\overline{H}(x)\| \ge \dfrac{\varepsilon \|x\|}{\|T\|}$. Therefore, $\|\overline{H}(x)\| \to +\infty$, i.e., $\|H(x)\| \to +\infty$ as $\|x\| \to +\infty$.

From steps 1 and 2, according to Lemma 3, we know that for every input u, map $H(x)$ is a homeomorphism on R^n, so system (1) has a unique equilibrium point. The proof is completed.

IV. Global exponential stability

Theorem 2. If Assumption 1 is satisfied and $D-(|B|+|\alpha|+|\beta|)L$ is an M-matrix, then for each u, system (1) has a unique equilibrium point, which is globally exponentially stable.

Proof. Since $D-(|B|+|\alpha|+|\beta|)L$ is an M-matrix, from Theorem 1, systems (1) have a unique equilibrium point x^*. Let $y(t)=x(t)-x^*$, (1) can be written as

$$\dot{y}_i(t) = -d_i(t)y_i(t) + \sum_{j=1}^{n} b_{ij}(t) f_j(y_j(t-\tau_{ij}(t)) + x_j^*) - \sum_{j=1}^{n} b_{ij}(t) f_j(x_j^*)$$
$$\bigwedge_{j=1}^{n} \alpha_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) f_j(y_j(s)+x_j^*)ds - \bigwedge_{j=1}^{n} \alpha_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) f_j(x_j^*)ds$$
$$\bigvee_{j=1}^{n} \beta_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) f_j(y_j(s)+x_j^*)ds - \bigvee_{j=1}^{n} \beta_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) f_j(x_j^*)ds,\ i=1,2,\cdots,n. \quad (8)$$

The initial conditions of equation (13) are $\Psi(s)=\phi(s)-x^*, s\in[-\tau,0]$. Systems (13) have a unique equilibrium at $y=0$.

Due to $D-(|B|+|\alpha|+|\beta|)L$ is an M-matrix, from the Lemma 1, we get that there exist positive constant numbers $\xi_i, i=1,2,\cdots n,$ satisfy

$$-\xi_i d_i + \sum_{j=1}^{n} \xi_j (|b_{ij}| + |\alpha_{ij}| + |\beta_{ij}|) L_j < 0,\ i=1,2,\cdots n. \quad (9)$$

Constructing function

$$F_i(\mu) = -\xi_i(d_i-\mu) + \sum_{j=1}^{n} \xi_j [e^{\mu\tau} |b_{ij}| + (|\alpha_{ij}| + |\beta_{ij}|) p_{ij}(\mu)] L_j,\ i=1,2,\cdots n.$$

It is obviously that $F_i(\mu)$ is a continuous function about μ and from (9) we know that

$$F_i(0) = -\xi_i d_i + \sum_{j=1}^{n} \xi_j (|b_{ij}| + |\alpha_{ij}| + |\beta_{ij}|) L_j < 0,\ i=1,2,\cdots n.$$

So, there exists a constant $\lambda > 0$ such that

$$-\xi_i(d_i-\lambda) + \sum_{j=1}^{n} \xi_j [e^{\lambda\tau} |b_{ij}| + (|\alpha_{ij}| + |\beta_{ij}|) p_{ij}(\lambda)] L_j < 0,\ i=1,2,\cdots n. \quad (10)$$

Let

$$V_i(t) = e^{\lambda t} |y_i(t)| \quad (11)$$

where λ is a constant to be given. Calculating the upper right derivative of $V_i(t)$ along the solutions of (8), from Assumption 1 ,Assumption 2 and Lemma 2, we get we have

$$D^+(V_i(t)) = e^{\lambda t}\operatorname{sgn}(y_i(t))[\dot{y}_i(t) + \lambda y_i(t)]$$
$$\le e^{\lambda t}\{-d_i(t)\ |y_i(t)| + \sum_{j=1}^{n} |b_{ij}(t)|\, |f_j(y_j(t-\tau_{ij}(t)) + x_j^*) - f_j(x_j^*)|$$

$$+|\bigwedge_{j=1}^{n}\alpha_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)f_j(y_j(s)+x_j^*)ds-\bigwedge_{j=1}^{n}\alpha_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)f_j(x_j^*)ds|$$

$$+|\bigvee_{j=1}^{n}\beta_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)f_j(y_j(s)+x_j^*)ds$$

$$-\bigvee_{j=1}^{n}\beta_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)f_j(x_j^*)ds|+\lambda|y_i(t)|\}$$

$$\le e^{\lambda t}\{-d_i\ |y_i(t)|$$

$$+\sum_{j=1}^{n}|b_{ij}||f_j(y_j(t-\tau_{ij}(t))+x_j^*)-f_j(x_j^*)|$$

$$+\sum_{j=1}^{n}|\alpha_{ij}||\int_{-\infty}^{t}k_{ij}(t-s)f_j(y_j(s)+x_j^*)ds-\int_{-\infty}^{t}k_{ij}(t-s)f_j(x_j^*)ds|$$

$$+\sum_{j=1}^{n}|\beta_{ij}||\int_{-\infty}^{t}k_{ij}(t-s)f_j(y_j(s)+x_j^*)ds-\int_{-\infty}^{t}k_{ij}(t-s)f_j(x_j^*)ds|+\lambda|y_i(t)|\}$$

$$\le e^{\lambda t}\{-d_i\ |y_i(t)|+\sum_{j=1}^{n}|b_{ij}|L_j|y_j(t-\tau_{ij}(t))|$$

$$+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)L_j\int_{-\infty}^{t}k_{ij}(t-s)|y_j(s)|ds+\lambda|y_i(t)|\}$$

$$\le -d_ie^{\lambda t}\ |y_i(t)|+\sum_{j=1}^{n}|b_{ij}|L_je^{\lambda\tau_{ij}(t)}e^{\lambda(t-\tau_{ij}(t))}|y_j(t-\tau_{ij}(t))|$$

$$+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)L_j\int_{-\infty}^{t}k_{ij}(t-s)e^{\lambda(t-s)}e^{\lambda s}|y_j(s)|ds+\lambda e^{\lambda t}|y_i(t)|$$

$$=(-d_i+\lambda)V_i(t)+\sum_{j=1}^{n}|b_{ij}|L_je^{\lambda\tau_{ij}(t)}V_j(t-\tau_{ij}(t))$$

$$+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)L_j\int_{-\infty}^{t}e^{\lambda(t-s)}k_{ij}(t-s)V_j(s)ds$$

$$\le(-d_i+\lambda)V_i(t)+\sum_{j=1}^{n}|b_{ij}|L_je^{\lambda\tau}\sup_{t-\tau\le s\le t}V_j(s)$$

$$+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)L_j\int_{-\infty}^{t}e^{\lambda(t-s)}k_{ij}(t-s)V_j(s)ds,\ i=1,2,\cdots,n. \tag{12}$$

Let $\xi_M=\max_{i=1,\cdots,n}\xi_i$, $\xi_m=\min_{i=1,\cdots,n}\xi_i$, taking $l_0=(1+\delta)e^{\lambda\tau}\,\|\Psi\|/\xi_m$, $\delta>0$ is a constant. Then,

$$V_i(s)=\mathrm{e}^{\lambda s}|\Psi_i(s)|<\xi_il_0,\ -\tau\le s\le 0,\ i=1,2,\cdots n. \tag{13}$$

In the following we prove

$$V_i(t)<\xi_il_0,\ \ t>0,\ i=1,2,\cdots n. \tag{14}$$

If (14) is not true, then from (13), there exist $t_1>0$ and some i such that

$$V_i(t_1)=\xi_il_0,\ \ D^+(V_i(t_1))\ge 0,\ V_j(t)\le\xi_jl_0,\ j=1,2,\cdots n,\ t\in[-\tau,t_1]. \tag{15}$$

According to (10), (12) we get

$$D^+\{V_i(t_1))\le(-d_i+\lambda)V_i(t_1)+\sum_{j=1}^{n}|b_{ij}|L_je^{\lambda\tau}\sup_{t-\tau\le s\le t_1}V_j(s)$$

$$+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)L_j\int_{-\infty}^{t_1}e^{\lambda(t_1-s)}k_{ij}(t_1-s)V_j(s)ds$$
$$\le \{-\xi_i(d_i-\lambda)+\sum_{j=1}^{n}\xi_j[e^{\lambda\tau}|b_{ij}|+(|\alpha_{ij}|+|\beta_{ij}|)\int_0^{+\infty}e^{\lambda\mu}k_{ij}(\mu)d\mu]L_j\}l_0$$
$$=\{-\xi_i(d_i-\lambda)+\sum_{j=1}^{n}\xi_j[e^{\lambda\tau}|b_{ij}|+(|\alpha_{ij}|+|\beta_{ij}|)p_{ij}(\lambda)]L_j\}l_0<0$$

However in (15), $D^+(V_i(t_1))\ge 0$, this is a contradiction. So $V_i(t)<\xi_i l_0$, for all $t>0$. Furthermore, from (11), (14), we get

$$|y_i(t)|\le \xi_i l_0 e^{-\lambda t}\le (1+\sigma)e^{\lambda\tau}\|\Psi\|\xi_M/\xi_m e^{-\lambda t}=M\|\Psi\|e^{-\lambda t},\ t\ge 0, i=1,2,\cdots n.$$

So $|x(t)-x^*|\le M\|\phi-x^*\|e^{-\lambda t}$, where $M=(1+\sigma)e^{\lambda\tau}\xi_M/\xi_m$. From the Definition 1, the equilibrium point of (1) is globally exponential stable. The proof is completed.

V. An illustrative example

Consider the two-dimensional neural networks with variable coefficients and unbounded delays, where

$$D(t)=\{d_{ij}(t)\}_{2\times 2}=\begin{bmatrix}3-\sin(t) & 0\\ 0 & 1\end{bmatrix}, B(t)=\{b_{ij}(t)\}_{2\times 2}=\begin{bmatrix}-0.3\sin(t) & -0.2\\ 0.12 & -0.2\cos(t)\end{bmatrix},$$
$$\alpha(t)=\{\alpha_{ij}(t)\}_{2\times 2}=\begin{bmatrix}0.1 & 0.3\cos(t)\\ 0.1\sin(t) & 0.1\end{bmatrix}, \beta(t)=\{\beta_{ij}(t)\}_{2\times 2}=\begin{bmatrix}0.13 & 0.2\cos(t)\\ 0.1 & 0.1\end{bmatrix};$$

$f_1(u)=\sin(u)$, $f_2(u)=(e^u-e^{-u})/(e^u+e^{-u})$; $k_1(t)=e^{-t}$, $k_2(t)=2/(1+t^3)$. It is easy to verify that $d_i(t)$, $b_{ij}(t)$, $\alpha_{ij}(t)$, $\beta_{ij}(t)$ satisfy Assumption 1; $f_1(u)$, $f_2(u)$ satisfy Assumption 2 with and $L_1=L_2=1$; $k_1(t)$ and $k_2(t)$ satisfy initial condition(A). Thus we get

$$D=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}, B=\begin{bmatrix}0.3 & -0.2\\ 0.12 & 0.2\end{bmatrix}, \alpha=\begin{bmatrix}0.1 & 0.3\\ 0.1 & 0.1\end{bmatrix}, \beta=\begin{bmatrix}0.13 & 0.2\\ 0.1 & 0.1\end{bmatrix}, L=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$ So we obtain

$D-(|B|+|\alpha|+|\beta|)L=\begin{bmatrix}1.47 & -0.7\\ -0.32 & 0.6\end{bmatrix}$ is a M-matrix, by Theorem 2, the neural network s is globally exponential stable.

VI. Conclusion

In this paper, without assuming the boundedness and differentiability of the activation functions, we analyze the existence, uniqueness, and globally exponential stability of the equilibrium point of fuzzy cellular neural networks with variable coefficients and unbounded delays. Applying the idea of Vector Liapunov function method and M-matrix theory, new criteria are derived for ascertaining existence, uniqueness for the equilibrium point and its global exponential stability of fuzzy cellular neural networks with variable coefficients and unbounded delays. In addition, due to sufficient conditions obtained are independent of the delays, these criteria can be easily checked in practice.

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